The Sieve of Eratosthenes

Eratosthenes of Cyrene (c. 276 BC – c. 195/194 BC) was a Greek mathematician, geographer, poet, astronomer, and music theorist. He served as chief librarian at the Library of Alexandria and is credited as having invented the discipline of geography, including creating the first map of the world incorporating parallels and meridians.

Eratosthenes is perhaps best known for being the first person to calculate the circumference of the Earth, which he did by comparing altitudes of the mid-day sun at two places a known North-South distance apart and for calculating the tilt of the Earth's axis, both with remarkable accuracy. Additionally, he may have accurately calculated the distance from the Earth to the Sun and invented the leap day.

Eratosthenes is also credited with the process known today as the Sieve of Eratosthenes. Use the chart below and the instructions on the next page to replicate Eratosthenes sieve.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Sieve of Eratosthenes

Put a box around the number 1. It is special, because it only has one factor.

Circle the number 2, then cross out every other multiple of 2. When you cross out the multiples of 2, cross through the square from lower left to upper rights – like . (Remember, the multiples of 2 are 2, 4, 6, 8, 10, 12, etc. – but don't cross out the 2.)

Before we continue with the Sieve of Eratosthenes, we are going to take a detour to **DIVISIBILITY RULES**. A divisibility rule is a rule that tells you whether you can divide a number by another number without leaving a remainder. For example, 14 is divisible by 2 because you can divide 14 by 2 and not have a remainder.

You probably already know the divisibility rule for 2. If you were asked, can you divide 1354 by 2 without a remainder, you most likely know that the answer is "yes." How do you know?

You can tell that 1354 is divisible by 2 just by looking at the digit in the one's place (or in this case, the digit 4).

The divisibility rule for 2 can be written…

A number is divisible by 2 if it ends in 0, 2, 4, 6, or 8.

Some people might say a number is divisible by 2 if it ends in an even number. This is correct, but isn't helpful if you don't know what an even number is. Take a look at the Sieve of Eratosthenes chart. Notice that all of the numbers you crossed out are in the columns that end in 0, 2, 4, 6, or 8.

Now, back to the Sieve of Eratosthenes.

Circle the number 3, then cross out every other multiple of 3. When you cross out the multiples of 3, cross through the square from upper left to lower rights – like or use a different color. (Remember, the multiples of 3 are 3, 6, 9, 12, 15, 18, etc. – but don't cross out the 3.)

The divisibility rule for 3 seems to be one of those "best kept secrets," which is really a shame, because it is really, really useful.

Can you tell whether 1354 is divisible by 3 just by looking at the number? If not, you are in for a treat!

A number is divisible by 3 if its digit sum (or the sum of all its digits) is divisible by 3.

For the number 1354, the digit sum is 1 + 3 + 5 + 4 or 13. Since 13 isn't divisible by 3 (you get a remainder when you divide 13 by 3), the number 1354 is not divisible by 3. However, the number 1356 is divisible by 3 because 1 + 3 + 5 + 6 = 15 and 15 is divisible by 3.

Practice using the divisibility rule for 3 on some random 5 and 6 digit numbers. You can always check to see if you are correct using a calculator.

BUT, what if the number is really large? Is 19,378,246,552,864,116 divisible by 3? If you use the divisibility rule for 3 and find the digit sum, 1 + 9 + 3 + 7 + 8 + 2 + 4 + 6 + 5 + 5 + 2 + 8 + 6 + 4 + 1 + 1 + 6 you get 78. (To make it easier to find the digit sum, notice that the digits are paired to make sets of 10, so 1 + 9 = 10, 3 + 7 = 10, 8 + 2 =10, and so on until the last three digits.) The problem is, do you know if 78 is divisible by 3? If you aren't certain, you can do the digit sum of the digit sum. The digit sum of 78 is 7 + 8 or 15. You know that 15 is divisible by three, but if you didn't, you could do another digit sum, so 1 + 5 = 6 which we know for certain is divisible by 3. So, we know that 19,378,246,552,864,116 is divisible by 3.

You probably won't have to tell whether a 17 digit number is divisible by 3 (except maybe on a math test or if you are a contestant on Jeopardy), but the divisibility rule is really handy to determine if 72 is divisible by 3. 7 + 2 = 9, so since 9 is divisible by 3, 72 is also divisible by 3.

Do a quick scan of the numbers you crossed out as multiples of 3 on the Sieve of Eratosthenes chart. Notice how they all have digit sums that are divisible by 3.

Continuing on the Sieve of Eratosthenes chart… Notice the numbers that you've crossed out twice. The numbers that were BOTH multiples of 2 AND multiples of 3 are 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, and 96. These are the multiples of 6. We can combine all of this information to generate the divisibility rule for 6.

A number is divisible by 6 if it is divisible by BOTH 2 AND 3.

Is 19,378,246,552,864,116 divisible by 6? Since we checked above that it was divisible by 3 (the digit sum is divisible by 3) and we can tell that it is divisible by 2 (since it ends in 6), we know that 19,378,246,552,864,116 is divisible by 6.

Returning once again to the Sieve of Eratosthenes chart… Circle the number 5 and cross out all of the multiples of 5 except 5 (10, 15, 20, 25, etc.). This is another divisibility rule that you most likely already know.

A number is divisible by 5 if it ends in 0 or 5.

Since our chart only goes up to 100, we only have one more step to go to complete Sieve of Eratosthenes. Circle the number 7 and cross out all of the multiples of 7 except 7. (The multiples of 7 that need to be crossed out are 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, and 98, but all but the 49, 77 and 91 have already been crossed out, so just make certain you cross out 49, 77, and 91.)

There is a divisibility rule for 7, but we aren't going to learn it here. The main purpose for learning the divisibility rules is because they make work easier and faster. The divisibility rule for 7 is complicated and can take just as long or longer to use than just dividing by 7. So, you don't need to know the divisibility rule for 7. If you are interested though, just do a web search and you can find several websites that show you how the divisibility rule works.

What does the Sieve of Eratosthenes do???? Make a list of all of the circled numbers from the first row and all of the numbers that didn't get crossed out in the rest of the chart.

List them here…

You should recognize these numbers as PRIME NUMBERS, or numbers that are only divisible by 1 and themselves.

Save this list. It may come in handy throughout the rest of this fraction review module.

By the way, your list should include 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97.

Continuing with the divisibility rules… We have divisibility rules for 2, 3, 5, and 6. Here are the divisibility rules for 4, 8, 9 and 10.

A number is divisible by 4 if the number formed by the last two digits is divisible by 4.

This rule is related to the divisibility rule for 2. For 2, we look to see if the last digit is divisible by 2. For this rule, we look at the number formed by the last two digits. For example, if the number is 3512, the number formed by the last two digits is 12. Since we know that 12 is divisible by 4, we know that 3512 will be divisible by 4.

A number is divisible by 8 if the number formed by the last three digits is divisible by 8.

This rule is an extension of the rules for 2 and 4 (and is even less useful). If you have a number like 51,832 where it is easier to see that 832 is divisible by 8, it can be a little handy.

The divisibility rule for 9 can be more useful. IT is very similar to the divisibility rule for 3.

A number is divisible by 9 if the digit sum is divisible by 9.

The number 82,557 is divisible by 9 because 8 + 2 + 5 + 5 + 7 = 27 and 27 is divisible by 9. Remember that if you don't know that 27 is divisible by 9 you can always take the digit sum of the digit sum and add 2 + 7 to get 9, which is divisible by 9.

The last divisibility rule is one you already know.

A number is divisible by 10 if it ends in a 0.

These divisibility rules (especially the ones for 2, 3, and 5) are critical tools for your fraction tool kit. Being able to say them and use them will greatly enhance your fluency with working with fractions. Spend time saying them, writing them and using them. The following worksheet will give you a chance to make a note page for future reference.

DIVISIBILITY RULES NOTES

	State the divisibility rule. Begin each statement with, "A number is divisible by ___ if…"	Is 91,827,364,557,312 divisible by the number? Explain why or why not.
2		
3		
4		
5		
6		
8		
9		
10		