SIMPLIFYING FRACTIONS (PART I) Name KEY

Simplifying can be pretty simple if the numbers are small and it is easy to see that both the numerator and the denominator can be divided by the same number.

For example, it is pretty easy to see that both the 15 and 20 in $\frac{15}{20}$ are divisible by 5, so

$\frac{15}{20} = \frac{15 \div 5}{20 \div 5} = \frac{3}{4}$.

This is where the divisibility rules come in handy.

Simplify $\frac{18}{54}$.

You may have done this problem differently, but one way to think about it is…

Using the divisibility rules it is easy to see that 18 and 54 are both divisible by 2. Using the digit sums, it is also easy to see that they are both divisible by 3. Since 18 and 54 are divisible by both 2 and 3, we know that they are both divisible by 6. So…

$\frac{18}{54} = \frac{18 \div 6}{54 \div 6} = \frac{3}{9}$ but notice, that $\frac{3}{9}$ isn't simplified completely because both 3 and 9 are divisible by 3.

Dividing both the numerator and the denominator by 3 we get…

$\frac{3}{9} = \frac{3 \div 3}{9 \div 3} = \frac{1}{3}$

If you recognized that 18 and 54 were both divisible by 18, you could have done this problem in one step, $\frac{18}{54} = \frac{18 \div 18}{54 \div 18} = \frac{1}{3}$, but it would probably take most people longer to figure out that 18 and 54 are both divisible by 18 than it takes to divide twice (or even three times).

Try this problem.

Simplify $\frac{48}{72}$

The worst case scenario is that you take 4 steps to simplify this fraction.

$$\frac{48}{72} = \frac{48 \div 2}{72 \div 2} = \frac{24}{36}$$
$$\frac{24}{36} = \frac{24 \div 2}{36 \div 2} = \frac{12}{18}$$
$$\frac{12}{18} = \frac{12 \div 2}{18 \div 2} = \frac{6}{9}$$
$$\frac{6}{9} = \frac{6 \div 3}{9 \div 3} = \frac{2}{3}$$

If you recognize that 48 and 72 are both divisible by 24, then you can simplify the fraction in one step.

$$\frac{48}{72} = \frac{48 \div 24}{72 \div 24} = \frac{2}{3}$$

The majority of the problems where you need to simplify fractions can be done using this method, but occasionally it isn't so easy to recognize a common factor for the numerator and denominator of a fractions.

What if the numbers are big, or you don't see a common factor for the numerator and denominator. Try simplifying this fraction.

$$\frac{210}{1764}$$

This is when a method known as simplifying fractions using prime factorization works really well.

The videos in the SIMPLIFYING FRACTIONS section of Unit 4 also describe the prime factorization method for simplifying fractions.

SIMPLIFYING FRACTIONS (Part II)

Simplifying Fractions Using Prime Factorization

We just simplified the fraction $\frac{48}{72}$ on the first page by dividing both the numerator and denominator by common factors. That is definitely a method that will work, but here's another method.

Begin first by finding the prime factorization of 48.

Now find the prime factorization of 72.

If you write the fraction $\frac{48}{72}$ using these prime factorizations you should notice the common factors in the numerator an denominator.

$$\frac{48}{72} = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3}$$

Crossing out a 2 in the numerator and in the denominator at the same time looks like this...

$\frac{48}{72} = \frac{\cancel{2} \cdot 2 \cdot 2 \cdot 2 \cdot 3}{\cancel{2} \cdot 2 \cdot 2 \cdot 3 \cdot 3} = \frac{24}{36}$ and does exactly the same thing as what we did above $\frac{48}{72} = \frac{48 \div 2}{72 \div 2} = \frac{24}{36}$

So... Once you have the fraction written using prime factorization, you can cross out all matching factors in the numerator and denominator.

$$\frac{48}{72} = \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 2 \cdot \cancel{3}}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{3} \cdot 3} = \frac{2}{3}$$

Whether or not this method is faster depends on the difficulty of the fraction and your fluency with prime factorization. If you've been doing the prime factorization by using factor trees or the inverted division method, try seeing if you can do prime factorizations off the top of your head.

Example... I know that 48 is the same as 6 x 8. 8 is 2 x 2 x 2 (you will use this a lot, so it is worth memorizing) and 6 is 2 x 3, so 48 is 2 x 2 x 2 x 2 x 3.

72 is 8 x 9 or 2 x 2 x 2 times 3 x 3 or 2 x 2 x 2 x 3 x 3. Thinking this through is much faster than working it out by writing it down.

Let's try another one.

Simplify $\frac{63}{84}$

63 is 7 x 9 and 9 is 3 x 3, so 63 is 3 x 3 x 7 (since 7 is already prime).

One way to think about 84 is 2 x 42 and 42 is 6 x 7 and 6 is 2 x 3, so 84 is 2 x 2 x 3 x 7.

$$\frac{63}{84} = \frac{3 \cdot 3 \cdot 7}{2 \cdot 2 \cdot 3 \cdot 7}$$

Crossing out (also known as factoring out) common factors in the numerator and denominator leaves

$\frac{63}{84} = \frac{\cancel{3} \cdot 3 \cdot \cancel{7}}{2 \cdot 2 \cdot \cancel{3} \cdot \cancel{7}} = \frac{3}{4}$ (Notice that the remaining $2 \cdot 2$ factors are just multiplied back together.)

If all of the factors get crossed out in either the numerator or denominator, that numerator or denominator becomes 1.

Let's go back to the fraction $\frac{210}{1764}$. Even though 210 and 1764 are larger numbers (and you may not be able to do a prime factorization without using inverted division or a factor tree), doing the prime factorization and simplifying the fraction using that method will probably be just as fast as dividing by a bunch of smaller numbers.

Try it here:

$\frac{210}{1764} = \frac{2 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 2 \cdot 3 \cdot 3 \cdot 7 \cdot 7} = \frac{5}{42}$

2|210
3|105
5|35
7

2|1764
2|882
3|441
3|147
7|49
7

Here's one more to try: $\frac{78}{104}$

$\frac{78}{104} = \frac{2 \cdot 3 \cdot 13}{2 \cdot 2 \cdot 2 \cdot 13}$

2|78
3|39
13

2|104
2|52
2|26
13

It is pretty easy to see that both the numerator and denominator of $\frac{78}{104}$ have a common factor of 2.

$\frac{78}{104} = \frac{78 \div 2}{104 \div 2} = \frac{39}{52}$ BUT… it isn't as obvious that 39 and 52 have a common factor other than 1.

Simplifying this fraction using prime factorization looks like this…

$\frac{78}{104} = \frac{2 \cdot 3 \cdot 13}{2 \cdot 2 \cdot 2 \cdot 13}$ which leads to $\frac{78}{104} = \frac{\cancel{2} \cdot 3 \cdot \cancel{13}}{\cancel{2} \cdot 2 \cdot 2 \cdot \cancel{13}} = \frac{3}{4}$

Practice simplifying the following fractions:

$\frac{48}{72} = \frac{2}{3}$ $\frac{16}{20} = \frac{4}{5}$ $\frac{25}{60} = \frac{5}{12}$ $\frac{72}{108} = \frac{2}{3}$

$\frac{84}{105} = \frac{4}{5}$ $\frac{28}{36} = \frac{7}{9}$ $\frac{102}{136} = \frac{3}{4}$ $\frac{16}{120} = \frac{2}{15}$