DIVISION OF FRACTIONS Name _____

We can write the problem 12 divided by 3 symbolically in three different ways.

$12 \div 3$ $3\overline{)12}$ $\frac{12}{3}$

If we have a fraction division problem we generally don't use the $\overline{)}$ symbol, but we do write it like…

$\frac{3}{4} \div \frac{5}{6}$ or $\frac{\frac{3}{4}}{\frac{5}{6}}$

If you aren't a fan of fractions to begin with, then you may not be impressed by a fraction containing fractions, but it is helpful for understanding what happens when we divide fractions.

A fraction where the numerator or denominator or both the numerator and denominator are fractions is called a complex fraction.

When you see $\frac{27}{4}$ you can interpret it in more than one way. If you think about it as a fraction, then you can multiply (or divide) the numerator and the denominator by the same number and get an EQUIVALENT FRACTION.

So… $\frac{27}{4} = \frac{27 \cdot 2}{4 \cdot 2} = \frac{54}{8}$ … etc.

If you treat $\frac{27}{4}$ as a division problem, like $27 \div 4$ you can get an equivalent fraction $6\frac{3}{4}$.

Note that $\frac{54}{8}$ from above is also $6\frac{3}{4}$.

We are going to combine these ideas to do fraction division problems.

If we write $\frac{3}{4} \div \frac{5}{6}$ as a complex fraction $\frac{\frac{3}{4}}{\frac{5}{6}}$, and think about it like any other fraction, we can multiply the numerator by anything as long as we multiply the denominator by the same amount.

What do you get if you multiply both the numerator and the denominator of the complex fraction by $\frac{6}{5}$?

$$\frac{\frac{3}{4} \cdot \frac{6}{5}}{\frac{5}{6} \cdot \frac{6}{5}} =$$

You should get $\frac{9}{10}$ $\left(\text{or } \frac{18}{20} \text{ if you didn't simplify}\right)$ because the denominator becomes 1.

$\frac{6}{5}$ is called the RECIPROCAL of $\frac{5}{6}$ because $\frac{6}{5} \cdot \frac{5}{6} = 1$. The product of any two reciprocals is always 1. Multiplying the denominator of the original complex fraction by its reciprocal guarantees that the numerator will become 1, leaving only the multiplication problem in the numerator.

$\frac{9}{10}$ is the answer to both the division problem $\frac{3}{4} \div \frac{5}{6}$ and the multiplication problem $\frac{3}{4} \cdot \frac{6}{5}$. By MULTIPLYING BY THE RECIPROCAL, the original division problem has been changed to a multiplication problem with the same answer.

You can follow this process to change any division of fractions problem to a multiplication of fractions problem that will have the same answer.

You might say that you don't need to learn how to DIVIDE FRACTIONS if you know how to change a division of fractions problem into a FRACTION MULTIPLICATION problem that will have the same answer.

To review… In order to divide fractions, rewrite the division problem into a multiplication problem with the same answer. Instead of dividing by the fraction, MULTIPLY BY ITS RECIPROCAL.

A division problem like $\frac{3}{8} \div \frac{5}{6}$ becomes $\frac{3}{8} \cdot \frac{6}{5}$, a multiplication problem with the same answer.

Rewrite each of these fraction division problems as a multiplication problem, and solve. (Remember that you will want to change mixed numbers to improper fractions first.)

$\frac{5}{8} \div \frac{5}{6}$

$\frac{21}{32} \div \frac{35}{48}$

$2\frac{7}{9} \div 2\frac{1}{12}$

$5\frac{5}{8} \div 1\frac{11}{25}$